Notes about the class:

Cover topics important for research in CM.

⇒ Start with DS's book, handouts of chapters from other book will be provided

⇒ Add: (1) Landau-Fermi liquid theory

(2) Luttinger-Tomonaga liquid

(3) Critical Phenomena and RG

(4) Monte Carlo methods

(5) Suggestions from students

Notation: Unless important, in this course we set:

$$1 = -1 = \pi = 2$$

HW: - every two weeks

- except last HW will take 4 weeks ⇒ MC for Ising model

Exams: mid-term and final, dates TBD

Grading: Everyone who tries gets an "A"

HW late by 1 week - 50%

HW late by more than 1 week - 0%

HW - 40% mid-term 30% final 30%

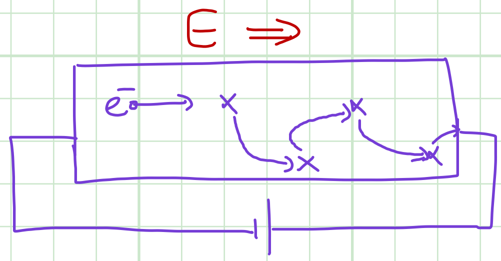
web: www.davidpekker.phyast.pitt.edu → Advanced Solid State class

HW1: due 20 Jan.

D.S. 5.22, 5.23, 7.1, 7.2

Why do metals (and semiconductors) have a finite resistance?

Short answer: scattering on impurities "slows down" electrons flowing in a metal



Goal: show how and when this mechanism works with simplest tools.

Boltzmann Equation:

Let us describe the distribution of electrons with the function

$n(r, k, t)$ — position, momentum, time

This is a semi-classical description that ignores the fact that position and momentum do not commute. [This is natural as Boltzmann did not know about quantum mechanics when he wrote down his eqn]

How does $n(r,k,t)$ depend on time?

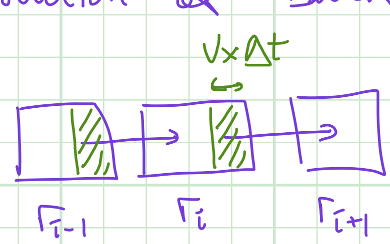
$$\frac{dn}{dt} = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial k_\mu}\frac{\partial k_\mu}{\partial t} + \frac{\partial n}{\partial r_\nu}\frac{\partial r_\nu}{\partial t}$$

What do these terms mean?

(1) $\frac{\partial n}{\partial r_\nu}\frac{\partial r_\nu}{\partial t}$ — particle motion ⇒ see by discretization

$\frac{\partial r}{\partial t} = \text{velocity} = \frac{\hbar k}{m}$ $\left[\text{for } \mathcal{H} = \frac{(\hbar k)^2}{2m}\right]$

Evolution of block at r_i: $n(r_i, k, t)$ ← density



particles leaving block at r_i in time Δt with momentum k?

⇒ momentum k ⇒ $v = \frac{\hbar k}{m}$

⇒ # leaving $= V \times \Delta t \times n(r, k, t)$

incoming particles $\Rightarrow n(r_{i-1},k,t)\frac{\hbar k}{m}\frac{\Delta t}{\Delta r}$

Outgoing particles $\Rightarrow n(r_i,k,t)\frac{\hbar k}{m}\frac{\Delta t}{\Delta r}$

Δn = net change $\Rightarrow \frac{[n(r_{i-1},k,t)-n(r_i,k,t)]}{\Delta r}\frac{\hbar k}{m}\Delta t = \frac{\partial}{\partial r_\nu} n(r,k,t)\frac{\hbar k_\nu}{m}\Delta t$

$$\Rightarrow \frac{dn}{dt} \sim \left[\partial_{r_\nu} n(r,k,t)\right]\frac{\hbar k_\nu}{m}$$

(2) $\frac{\partial n}{\partial k_\mu}\left(\frac{\partial k_\mu}{\partial t}\right) = \frac{\partial n}{\partial k_\mu}F_\mu \Rightarrow$ Response to outside forces like electric fields.

if $v = \frac{\hbar k}{m} \Rightarrow \frac{\partial k}{\partial t} = \frac{\partial}{\partial t}\frac{mv}{\hbar} = \frac{m}{\hbar}a = \frac{\cancel{m}\vec{F}}{\hbar\cancel{m}} = \frac{\vec{F}}{\hbar}$ $\qquad \vec{F} = m\vec{a} \underset{\text{e.g.}}{=} -e\vec{E} - \frac{e}{c}\vec{J}\times\vec{H}$

(3) $\frac{\partial n}{\partial t} = \left.\frac{\partial n}{\partial t}\right|_{ext} + \left.\frac{\partial n}{\partial t}\right|_{collision}$

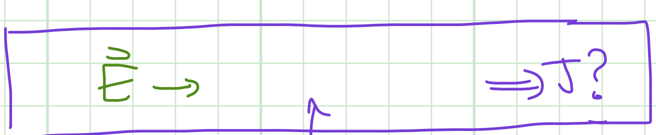
often called the "collision integral"

this term represents change of the distribution due to either external processes like particle loss or internal processes → the collisions between particles.

$$\frac{dn}{dt} = \left.\frac{\partial n}{\partial t}\right|_{ext} + \frac{\partial n}{\partial k_\mu}\frac{F_\mu}{\hbar} + \frac{\partial n}{\partial r_\nu}\frac{\hbar k_\nu}{m} + \left.\frac{\partial n}{\partial t}\right|_{collision} \iff \text{Boltzmann equation.}$$

How to use the Boltzmann eq?

Example: steady state current through a metal wire.



metal with Fermi-Dirac distribution

Goal: compute $\sigma = J/E$.

"Boundry conditions"

(1) steady state $\Rightarrow \frac{dn}{dt} = 0 \Rightarrow n(r,k,t) = n(r,k)$

(2) assume uniform distribution in space (no edges) $\Rightarrow n(r,k) = n(k)$

(3) We work away from the ends of the wire, where current is being injected/extracted $\Rightarrow \left.\frac{\partial n}{\partial t}\right|_{ext} = 0$.

Using these conditions the Boltzmann equation becomes:

$$\frac{dn}{dt} = 0 = -\frac{e\vec{E}_\mu}{\hbar}\frac{\partial n(k)}{\partial k_\mu} + \frac{\partial n}{\partial t}\Big|_{\text{collision}}$$

To make further progress we need to specify the collision integral.

(1) what should $n(k)$ be with no external Electric field?

$$n(k) = n_F(k) = \frac{1}{e^{\beta\varepsilon_k}+1} \Rightarrow \text{Fermi-Dirac distribution}$$

(2) How does this distribution get established?

$\Rightarrow$ collisions of electrons with impurities, phonons, each other bring the electrons to thermal equilibrium

$\hookrightarrow$ details of how this works are still a subject of active research, we will say a bit more about this later in the lecture.

$\Rightarrow$ Instead of describing the e^- thermalization explicitly we make the relaxation time approximation:

thermalization processes tend to drive $n(k)$ to its equilibrium form $n_F(k)$ in time $\tau(k)$

$$\Rightarrow \frac{\partial n}{\partial t} \approx \frac{n_F(k) - n(k)}{\tau(k)}$$

relaxation time approximation.

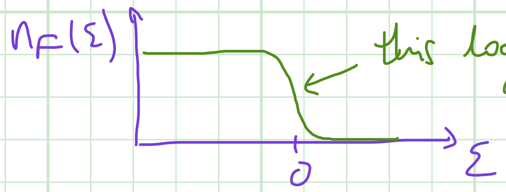
$$\frac{eE}{\hbar}\frac{\partial n}{\partial k} = \frac{n_F(k) - n(k)}{\tau} \Rightarrow n(k) = n_F(k) - \frac{\tau e E}{\hbar}\frac{\partial n}{\partial k}$$

This equation can be solved itteratively, at least for the case of small $\vec{E}$.

$$n^{(0)}(k) = n_F(k) \qquad n^{(1)}(k) = n_F(k) - \frac{\tau e E}{\hbar}\frac{\partial}{\partial k} n_F(k)$$

$$\frac{\partial}{\partial k} n_F(k) = \frac{\partial n_F(\varepsilon)}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial k} \simeq \delta(\varepsilon)\frac{\partial \varepsilon}{\partial k}$$

(1) Why $\frac{\partial n_F(\varepsilon)}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left[\frac{1}{e^{\beta\varepsilon}+1}\right] \simeq \delta(\varepsilon)$?



Indeed, consider

$$\int_{-\Lambda}^{\Lambda} d\varepsilon \, \frac{\partial n_F(\varepsilon)}{\partial \varepsilon} = n_F(\varepsilon)\Big|_{-\Lambda}^{\Lambda} = 1 \qquad \text{for } \Lambda \gg k_B T$$

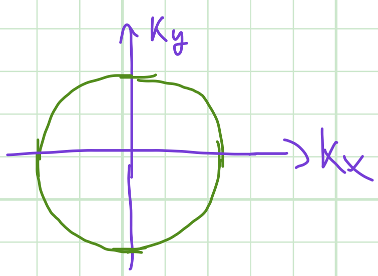
(2) Hence we find

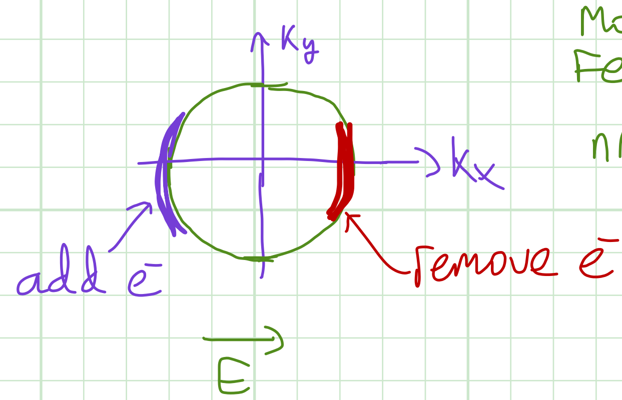
$$n^{(1)}(k) = n^F(k) - \frac{\tau e E_\mu}{\hbar}\,\delta(\varepsilon_k)\,\underbrace{\frac{\partial \varepsilon_k}{\partial k_\mu}}_{\frac{\hbar^2 k_\mu}{m}} \qquad \text{where } \varepsilon_k = \frac{\hbar^2 k^2}{2m} - \varepsilon_F$$

$$= n^F(k) - \frac{\tau e \hbar}{m}\, E\cdot k \,\delta(\varepsilon_k)$$

What does this mean?

(3) n(k) no E-field n(k), apply E-field





Modifications near Fermi surface

n(k) shifts so it is centered not on k=0 but on finite k.

(4) Total current and conductivity

$$\text{current density} = J_\alpha = -e\int \underbrace{\frac{\hbar k_\alpha}{m}}_{\text{Velocity}}\, n(k)\, \frac{d^3k}{(2\pi)^3} = -e\int \frac{\hbar k_\alpha}{m}\left[-\frac{\tau e \hbar}{m}\, E\cdot k\; \delta(\varepsilon_k)\right]\frac{d^3k}{(2\pi)^3}$$

$$J_x = \frac{E\, e^2 \tau \hbar^2}{m^2}\int \frac{k^2 dk\; d\cos\theta}{(2\pi)^2}\left[(k\cos\theta)^2\; \delta\!\left(\frac{\hbar^2 k^2}{2m} - \varepsilon_F\right)\right]$$

$$= \frac{E e^2 \tau \hbar^2}{m^2} \int \frac{2}{3} \frac{k^4\, dk}{(2\pi)^2}\, \delta\!\left(\frac{\hbar k^2}{2m} - \varepsilon_F\right)$$

$$\int_{-1}^{1} dz\, z^2$$

$$\frac{\hbar^2 k^2}{2m} = \varepsilon \;\Rightarrow\; \frac{\hbar^2 k\, dk}{m} = d\varepsilon \,, \quad k = \sqrt{2m\,\varepsilon/\hbar^2}$$

$$k\, dk = \frac{m\, d\varepsilon}{\hbar^2}$$

$$= \frac{2}{3} \frac{E e^2 \tau \cancel{\hbar^2}}{(2\pi)^2\, m^{\cancel{2}}} \int \left(\frac{2m\varepsilon}{\hbar^2}\right)^{3/2} \frac{\cancel{m}\, d\varepsilon}{\cancel{\hbar^2}}\, \delta(\varepsilon - \varepsilon_F)$$

$$= \frac{\cancel{2}}{3} \frac{E e^2 \tau}{\cancel{4}\pi^2 \cancel{m}} \frac{\cancel{2}\cancel{m}\,\varepsilon_F}{\hbar^2} \sqrt{\frac{2m\varepsilon_F}{\hbar^2}} = \frac{2\, E\, e^2 \tau\, 2m \sqrt{2m}\, \varepsilon_F^{3/2}}{2m\; 2\, 3\pi^2\, \hbar^3} = \boxed{\frac{E e^2 \tau\, n_0}{m}}$$

use: $$n_0 = \int_0^{k_F} \frac{d^3k}{(2\pi)^3} = \frac{4}{3} \frac{k_F^3}{8\pi^2} = \frac{k_F^3}{6\pi^2} = \frac{2m\, \varepsilon_F^{3/2} \sqrt{2m}}{\hbar^3\, 6\pi^2}$$

$$\boxed{\sigma = J/E = \frac{e^2 \tau\, n_0}{m}}$$ ⇐ Drude Formula! Using the Fermi-Dirac distribution has not changed the result.

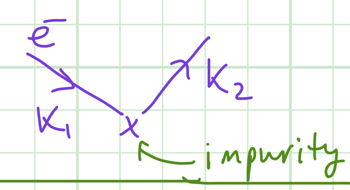
Note 1: The expression we obtained is a form of Linear Response. The current generated is proportional to the Electric field applied.

Note 2: what is going on "under the hood"

consider $E(t) = E_0\, \Theta(t)$

elastic scattering

$|k_1| = |k_2|$



(1) $t < 0$, $E(t) = 0$, $n(k) = n_F(k)$

(2) $t = 0$, $E(t) = E_0$, e^- start to accelerate

(3) $t \sim \tau$, Elastic scattering ~~of~~ e^- on impurities starts to take place and limits e^- velocity.

(4) $t = \tau_t$, e^- thermalize due to $e^- - e^-$ and $e^- - ph$ scattering. Typically $\tau_t \gg \tau$, so our relaxation time approximation is not justified: depending on response we want we should use a different relaxation time.

Where is the energy relaxation? This must occur in the last step.

Note 3: Aother way to state the result:

$$v_{drift} = \frac{J}{n_0 e} = \frac{e\tau}{m} = \tau \vec{F}$$